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Technical Comments

Brief discussion of previous investigations in the aerospace sciences and technical comments on papers published in the Journal of Guidance, Control, and Dynamics *are presented in this special department. Entries must be restricted to a maximum of 1000 words, or the equivalent of one* Journal *page including formulas and figures. A discussion will be published as quickly as possible after receipt of the manuscript. Neither the AIAA nor its editors are responsible for the opinions expressed by the correspondents. Authors will be invited to reply promptly.*

Comments on "Literal Approximations to Aircraft Dynamics Modes" and "Consistent Approximations to Aircraft Longitudinal Modes"

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Introduction

LINEARIZED aircraft dynamics have been studied for nearly 100 years[1−3] and have been standardized since the 1930s through the work of Melvill Jones.[4] Since then, the equations of motion have not been modified and may be found in any textbook dealing with aircraft stability and control. Nevertheless, the problem of finding literal approximations to aircraft transfer functions, and in particular to aircraft dynamic modes, still continues to attract the attention of the aeronautical community. This is because a literal expression involving aerodynamic derivatives and inertial data provides physical insight into the stability of the aircraft motion. However, it is hardly possible to find exact closed-form solutions to the aircraft modes because both the longitudinal and the lateral aircraft eigenvalues come from fourth-order characteristic equations. This motivates interest in getting improved literal solutions and explains why new expressions appear from time to time in the literature. The writer's view about this issue is that it is unlikely that at this stage something significant can be added about such an old problem. For these reasons, in the following I revisit papers by Ananthkrishnan and Unnikrishnan[5] and Ananthkrishnan and Ramadevi[6] and indicate what is really new and what is not in them.

Discussion

The first paper[5] looks ambitious in that the authors claim 1) to follow, for the first time, a formal procedure for deriving literal approximations to aircraft dynamic modes, 2) to derive formally a single literal approximation to each mode, 3) to point out a major flaw in previous derivations that was responsible for the poor approximations of phugoid and dutch-roll modes, 4) to improve the slow mode approximations, 5) to recover satisfactory existing literal approximations for the fast modes, and 6) to derive results that are of fundamental nature and that are not particular to any class of airplanes or any particular set of aircraft data. As will be shown, the statements should be revised.

A fundamental reason for doing so is that the authors are not aware of the work by McRuer et al.,[7] which contains the most exhaustive collection of literal approximations to aircraft dynamics. In the following the authors' claims vis-a-vis the existing approximations by McRuer et al. are commented.

Start with the equations of motion. As usual, longitudinal and lateral dynamics are treated as two sets of decoupled differential equations and are summarized in Eqs. (2) and (3) of Ref. 5. Although the authors say that the dimensional stability derivatives are defined in the *standard* manner, some confusion arises because in Eqs. (3) the primed derivatives are not included as they should be, for example, see McRuer et al.[7] p. 257. This is probably due to a simplified model with $I_{xz} = 0$ or to the very unusual assumption that the body axes coincide with the principal axes of the aircraft.

The situation is further complicated by the introduction of derivatives with apex ($N'_\beta = N_\beta \cos\alpha_0 - L_\beta \sin\alpha_0$, etc.) that are not the usual primed derivatives (as the notation would suggest). The results of the paper about the lateral dynamics, for example, Eqs. (20–22), may be converted to a classical form by simply substituting the stability derivatives with the standard primed derivatives. Henceforth such a substitution is assumed to be made.

A more substantial problem about the longitudinal equations is the lack of the $M_{\dot\alpha}$ term in the pitching motion equation. The authors[5] state that "for convenience, derivatives with respect to $\dot\alpha$ are not considered." Unfortunately, this implies that, for mathematical tractability, an important physical phenomenon is neglected. Indeed, it is well known that the derivative $M_{\dot\alpha}$ is important in longitudinal dynamics because "it does have a significant, if not powerful, effect on the damping of the short period mode" (McRuer et al.[7] p. 275). With their approximation Ananthkrishnan and Unnikrishnan[5] get the following expression for the short period damping:

$$2\zeta_{SP}\omega_{SP} = -(M_q + Z_\alpha / V_0) \tag{1}$$

which is *not* the standard approximation, as they claim. In fact the standard approximation is (see McRuer et al.[7] p. 309, Stevens and Lewis[8] p. 207, or McLean[9] p. 79)

$$2\zeta_{SP}\omega_{SP} = -(M_q + Z_\alpha / V_0 + M_{\dot\alpha}) \tag{2}$$

and differs from Eq. (1) because of the term $M_{\dot\alpha}$. In practice, the importance of $M_{\dot\alpha}$ is remarkable. This is illustrated by numerical results shown in Table 1, where a number of aircraft data taken from the literature[10] are compared. It is seen that the mean percent error incurred with Eq. (1) is on the order of 15%. The short period approximation is always underestimated due to the lack of the damping term $M_{\dot\alpha}$.

As far as the phugoid mode is concerned, Ananthkrishnan and Unnikrishnan[5] seem to ignore approximations other than that (very crude) by Lanchester. Indeed they claim that "previous literal approximations... assumed $\Delta\alpha$ and $\Delta\dot\alpha$ to be zero during the phugoid mode." However this is not true if a three-degree-of-freedom phugoid model is considered (McRuer et al.[7] pp. 309–312). With such a model the pitch rate equation is not dropped and is approximated by

$$M_u \Delta V + M_\alpha \Delta\alpha \cong 0 \tag{3}$$

which clearly shows that $\Delta\alpha$ varies (slowly) due to the presence of $M_u \neq 0$. Also note that the literal approximation of phugoid natural frequency "discovered" by Ananthkrishnan and Unnikrishnan[5] is exactly the same suggested by McRuer et al.[7] p. 336. Further comments on the phugoid mode approximations are postponed.

To better analyze the literal approximations of lateral dynamics, consider the standard factorization of the characteristic polynomial

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Table 1 Comparison between different approximations of short period damping

Flight Condition[d]	$2\zeta_{SP}\omega_{SP}$,[a] rad/s	% error e_1[b]	% error e_2[c]
	F104		
3	4.9017	0.0214	−14.5113
4	2.6193	0.4105	−17.0238
5	0.6216	0.2910	−28.2767
6	1.3091	0.4994	−18.7236
7	1.4415	−0.3208	−10.7788
	Boeing 747		
3	1.5696	−1.6559	−8.7343
4	2.0843	−1.4740	−9.4159
5	0.9241	−1.0499	−8.1065
6	1.1867	−0.8981	−9.7100
7	1.4776	−0.6506	−12.5593
	Convair 880M		
3	1.4069	0.5299	−14.3086
4	2.0999	0.5684	−15.3930
5	1.0906	0.5561	−14.2728
6	1.2491	0.4087	−14.4209
7	1.3584	0.4331	−14.5153

[a]Exact value.
[b]Percent error with Eq. (2) from McRuer et al.[7]
[c]Percent error with Eq. (1) from Ananthkrishnan and Unnikrishnan.[5]
[d]Data and flight conditions from Heffley and Jewell.[10]

$$P_{\text{lat}}(\lambda) = \left(\lambda^2 + 2\zeta_{\text{DR}}\omega_{\text{DR}}\lambda + \omega_{\text{DR}}^2\right)(\lambda - \lambda_R)(\lambda - \lambda_S) \tag{4}$$

[See Eq. (19) of Ref. 5 where a sign error has been fixed.] Letting

$$P_{\text{lat}}(\lambda) = A\lambda^4 + B\lambda^3 + C\lambda^2 + D\lambda + E \tag{5}$$

one has $A = 1$ and

$$B = -\lambda_R - \lambda_S + 2\zeta_{\text{DR}}\omega_{\text{DR}} \tag{6}$$

$$C = \omega_{\text{DR}}^2 - 2\zeta_{\text{DR}}\omega_{\text{DR}}(\lambda_R + \lambda_S) + \lambda_R\lambda_S \tag{7}$$

$$D = -(\lambda_R + \lambda_S)\omega_{\text{DR}}^2 + 2\zeta_{\text{DR}}\omega_{\text{DR}}\lambda_R\lambda_S \tag{8}$$

$$E = \lambda_R\lambda_S\omega_{\text{DR}}^2 \tag{9}$$

Note that it is possible to evaluate the polynomial coefficients B, C, D and E as a function of the aerodynamic derivatives (Ref. 7, p. 358): The coefficients of the fourth-order lateral characteristic polynomial are

$$B = -L'_p - N'_r - Y_\beta/V_0 \tag{10}$$

$$C = N'_\beta + L'_p(N'_r + Y_\beta/V_0) - N'_p L'_r + Y_\beta N'_r/V_0 \tag{11}$$

$$D = -L'_p(N'_\beta + Y_\beta N'_r/V_0) + N'_p(L'_\beta + Y_\beta L'_r/V_0) - L'_\beta g/V_0 \tag{12}$$

$$E = (L'_\beta N'_r - N'_\beta L'_r)g/V_0 \tag{13}$$

First consider the spiral mode approximation. Because the spiral is a first-order mode with a long time constant, it is customary to assume $|\lambda_S| \ll \min(|\lambda_R|, \omega_{\text{DR}})$, from which $D \cong -\lambda_R\omega_{\text{DR}}^2$ and

$$\lambda_S \cong -(E/D) \tag{14}$$

Quoting McRuer et al., p. 378, "the approximation for the root (i.e. E/D) has been well known for many years." Ananthkrishnan and Unnikrishnan[5] claim that a new approximation for the spiral mode is given by Eq. (22) of Ref. 5. Unfortunately, their result is not new at all. Indeed, with the aid of Eqs. (12) and (13), it is easily checked that the expression found by Ananthkrishnan and Unnikrishnan coincides with $-E/D$. This is confirmed by that (with the notation in their paper, where left-hand side is LHS) their results may be expressed as $\lambda_S = \text{LHS}[S^1_{\text{lat}}]/\text{LHS}[S^2_{\text{lat}}]$, where $\text{LHS}[S^1_{\text{lat}}] \equiv E$ and $\text{LHS}[S^2_{\text{lat}}] \equiv -D$ [see Eqs. (8) and (9) of Ref. 5].

Also, the authors[5] suggest the following simplified formula for the spiral eigenvalue in stability axes:

$$\lambda_S = \frac{(g/V_0)(L'_\beta N'_r - N'_\beta L'_r)}{N'_\beta L'_p - L'_\beta N'_p} \tag{15}$$

However, this is a mere simplified version of the well-known spiral approximation

$$\lambda_S = \frac{(g/V_0)(L'_\beta N'_r - N'_\beta L'_r)}{N'_\beta L'_p - L'_\beta(N'_p - g/V_0)} \tag{16}$$

that is found by considering a spiral-roll subsidence model (McRuer et al.,[7] pp. 374–376 and Stevens and Lewis,[8] pp. 216–217).

We now turn our attention to the dutch-roll approximation. As already stated, we know that $D \cong -\lambda_R\omega_{\text{DR}}^2$. Hence,

$$\omega_{\text{DR}}^2 \cong -D/\lambda_R \tag{17}$$

which is the starting point of Ananthkrishnan and Unnikrishnan.[5] From the preceding equation and using that $\lambda_R \approx L'_p$, the authors derive the approximation for the dutch-roll natural frequency [see Eq. (20) of Ref. 5]. In the same Eq. (20), the dutch-roll damping is approximated as $2\zeta_{\text{DR}}\omega_{\text{DR}} = -N'_r - Y_\beta/V_0$. Now Ananthkrishnan and Unnikrishnan conclude that "although L_β has long been recognized as one of the more significant derivatives affecting the dutch-roll dynamics, the present derivation is the first to successfully capture this influence. Notably, Eq. (20) also captures the effect of gravity on the dutch-roll dynamics, which was missing in previous literal approximations." The preceeding quotation is incorrect for many reasons. From a physical viewpoint, it is clear that L_β should have a negligible effect on the dutch-roll natural frequency and a significant effect on the dutch-roll damping, but these properties are not reflected in Eqs. (20) of Ref. 5. The reason is that when computing ω_{DR} through $\omega_{\text{DR}}^2 = -D/\lambda_R$ an accurate approximation for λ_R is needed, whereas $\lambda_R = L'_p$ is not. A better approximation is available in the literature and is found, once again, with the spiral-roll subsidence model.[7,8] The result is

$$-\lambda_R = -L'_p - (L'_\beta/N'_\beta)[N'_p - (g/V_0)] \tag{18}$$

note that [see Eq. (12)]

$$D = -L'_p(N'_\beta + Y_\beta N'_r/V_0) + N'_p(L'_\beta + Y_\beta L'_r/V_0) - L'_\beta(g/V_0) \tag{19}$$

$$D \cong -L'_p(N'_\beta + Y_\beta N'_r/V_0) + L'_\beta[N'_p - (g/V_0)] \tag{20}$$

because, as usual, $|L'_\beta| \gg |Y_\beta L'_r|/V_0$. Accordingly, one has

$$D \cong \underbrace{(N'_\beta + Y_\beta N'_r/V_0)}_{\omega_{\text{DR}}^2}\underbrace{\left[-L'_p + \frac{L'_\beta}{N'_\beta + Y_\beta N'_r/V_0}\left(N'_p - \frac{g}{V_0}\right)\right]}_{\approx -\lambda_R} \tag{21}$$

where the first term is the classical approximation for the dutch-roll natural frequency and the second is very close to the roll pole given by Eq. (18). Thus, the validity of Eq. (17) is confirmed. To further validate this point, in Table 2 the classical dutch-roll approximation of natural frequency is compared to that proposed by Ananthkrishnan and Unnikrishnan[5] for a number of different cases. The superiority of the classical result is apparent.

Note that although the classical expression of ω_{DR}^2 does not contain the derivative L'_β, the latter *does* affect the dutch-roll dynamics. This occurs through the term $2\zeta_{\text{DR}}\omega_{\text{DR}}$. Indeed, it is well known that the two-degree-of-freedom approximation $2\zeta_{\text{DR}}\omega_{\text{DR}} \cong -N'_r - Y_\beta/V_0$ is a poor estimate of the dutch-roll damping (McRuer et al.[7] p. 379). A better result is given by

$$2\zeta_{\text{DR}}\omega_{\text{DR}} \cong -N'_r - (Y_\beta/V_0) - (L'_\beta/N'_\beta)[N'_p - (g/V_0)] \tag{22}$$

Note that this approximation contains both L'_β and the effect of gravity. Also, in contrast to the result of Ananthkrishnan and Unnikrishnan,[5] it does capture the possibility of divergent dutch-roll oscillations (when the term proportional to L'_β is large enough to overpower the two-degree-of-freedom damping).

Let us now turn our attention to the companion paper by Ananthkrishnan and Ramadevi,[6] where new approximations to aircraft longitudinal modes are sought. As in the first paper,[5] the equations of motion lack the important contribution of $M_{\dot\alpha}$. In addition, there are other fundamental points that must be considered. The paper[6]

Table 2 Comparison between different approximations of Dutch-roll natural frequency

Flight Condition[d]	ω_{DR},[a] rad/s	% error e_1[b]	% error e_2[c]
	F104		
3	7.5341	−6.1807	6.0136
4	4.5022	2.4603	−4.6422
5	2.8352	−0.2565	72.9157
6	2.8480	−3.5744	9.1371
7	4.2933	−1.1982	5.2426
	Boeing 747		
3	1.0598	−4.7422	6.4458
4	1.4055	−2.2453	3.8866
5	0.8628	−4.8583	11.3754
6	1.0740	−3.5692	8.1469
7	1.3074	−1.8178	4.8334
	Convair 880M		
3	1.4090	0.1915	1.4166
4	1.8802	0.2575	1.2418
5	1.6081	−16.6350	27.1599
6	1.4309	0.1072	1.6978
7	1.5398	0.2068	1.2601

[a]Exact value.
[b]Percent error with classical dutch-roll approximation (from McRuer et al.[7]).
[c]Percent error with approximation by Ananthkrishnan and Unnikrishnan.[5]
[d]Data and flight conditions from Heffley and Jewell.[10]

exploits the different order of magnitudes in the coefficients of the characteristic polynomial and in the equations of motion.

A first problem arises just at the beginning when the authors introduce an unusual shorthand notation for bookkeeping the nondimensional terms associated with the dimensional stability derivatives. They[6] say that "the precise expressions for these terms are not of interest because they will be replaced with the corresponding dimensional derivatives when the final results are presented." However, it is probably due to this lack of precision that fundamental misunderstandings are incurred about the relative importance of some terms. For instance, the expression

$$\sqrt{(\bar{q}Sc/I_y)}m_q + (g/V_0)z_\alpha \tag{23}$$

is translated [see Eq. (9) of Ref. 6] into something like $m_q + \epsilon z_\alpha$, where ϵ is a small parameter, wherefrom they conclude that the two terms are of different order of magnitude. As a consequence, the derivative M_q in Eq. (A3) of Ref. 6 is found of order 0 and the terms X_u and Z_α/V_0 both of order 1. As will be shown soon, this is wrong and is the cause of a lot of confusion. Indeed, it is a simple matter to check that M_q and Z_α/V_0 are of the same order and that Z_α/V_0 and X_u are not. To show this, recall that

$$Z_\alpha = -\left(\rho SV_0^2/2m\right)\left(C_{L_\alpha} + C_D\right) \cong -\left(\rho SV_0^2/2m\right)C_{L_\alpha} \tag{24}$$

Assuming trim conditions ($mg = \rho SV_0^2 C_L/2$), one gets

$$Z_\alpha/V_0 \cong -(g/V_0)C_{L_\alpha}/C_L \tag{25}$$

Also, the derivative M_q is defined as

$$M_q = \left(\rho SV_0^2 c^2/4I_y\right)C_{M_q} \tag{26}$$

For a conventional aircraft, the most important contribution to C_{M_q} is given by the horizontal tail. This can be estimated as

$$C_{M_q} \cong -2V_H(l_t/c)C_{L_{\alpha t}} \tag{27}$$

where, as usual, V_H is the horizontal tail volume coefficient, $C_{L_{\alpha t}}$ is the tail lift-curve slope, and l_t is the distance between aerodynamic centers of wing–body and tail. Invoking trim conditions, one has

$$M_q \cong -(g/V_0)C_{L_\alpha}/C_L\left[\left(C_{L_{\alpha t}}/C_{L_\alpha}\right)V_H\left(cl_t/\rho_y^2\right)\right] \tag{28}$$

where the radius of gyration ρ_y has been introduced. Finally, we get

$$[M_q/(Z_\alpha/V_0)] \cong \left(C_{L_{\alpha t}}/C_{L_\alpha}\right)V_H\left(cl_t/\rho_y^2\right) \tag{29}$$

which clearly shows that, for conventional airplanes, the ratio $M_q/(Z_\alpha/V_0)$ is about unity. This is confirmed by Table 3 where

Table 3 Comparison of aerodynamic derivatives in stability axes

Flight Condition[a]	M_q, s^{-1}	Z_α/V_0, s^{-1}	$M_q/(Z_\alpha/V_0)$	(M_u/M_α) $[(X_\alpha - g)/(Z_\alpha/V_0)]$
		F104		
3	−1.8700	−2.3204	0.8059	−0.0004
4	−0.9560	−1.2174	0.7853	−0.0079
5	−0.2200	−0.2259	0.9740	0.0253
6	−0.4340	−0.6300	0.6889	−0.0110
7	−0.4930	−0.7931	0.6216	0.0065
		Boeing 747		
3	−0.6990	−0.7335	0.9529	−0.0001
4	−0.9250	−0.9630	0.9605	0.0008
5	−0.4210	−0.4282	0.9832	−0.0021
6	−0.5350	−0.5365	0.9973	−0.0001
7	−0.6680	−0.6240	1.0705	0.0029
		Convair 880M		
3	−0.5780	−0.6276	0.9210	0.0003
4	−0.8500	−0.9267	0.9172	0.0001
5	−0.4310	−0.5040	0.8552	0.0007
6	−0.4930	−0.5759	0.8560	0.0002
7	−0.5300	−0.6312	0.8396	0.0002

[a]Aircraft data and flight conditions from Heffley and Jewell.[10]

different aircraft configurations are compared. In particular, in all cases but one Z_α/V_0 is greater than M_q, which is in sharp contrast to the view of Ananthkrishnan and Ramadevi.[6] In a similar way, it is a simple matter to check that

$$X_u \cong -(2g/V_0)C_D/C_L \tag{30}$$

and, accordingly,

$$[(Z_\alpha/V_0)/X_u] \cong (C_{L_\alpha}/2C_D) \gg 1 \tag{31}$$

which shows how Z_α/V_0 and X_u are not of the same order of magnitude.

Finally, the authors discover a third unconventional term that in their formulas is summed to both the short period natural frequency and damping. In particular, the expression for the short period damping is[6]

$$2\zeta_{SP}\omega_{SP} = -M_q - (Z_\alpha/V_0) - (M_u/M_\alpha)(X_\alpha - g) \tag{32}$$

Now, Ananthkrishnan and Ramadevi claim that "the third term happens to be of the same order as the second term." This is hardly believable. Indeed, there are *several* orders of magnitude between the two terms. This is definitively confirmed by the results in the last column of Table 3.

From the reasons just given, it appears that all of the conclusions of the authors[6] about the phugoid approximation must be carefully revised. This is confirmed by a numerical analysis of aircraft data. It can be checked that the phugoid frequency approximation given in Eq. (26) of Ref. 6 produces results coincident (from an engineering viewpoint) with those found with the McRuer et al. formula (Ref. 7 p. 336). Once again, this is due to an incorrect estimate of the order of magnitude of the additional term in the denominator of Eq. (26).

Conclusions

Aerodynamic derivatives are not mere mathematical symbols and it is impossible to obtain significant expressions without taking into account the relative importance of the different terms in the literal approximations. Actually, the precise knowledge of the order of magnitude of the stability derivatives in the equations of motion (and hence in the literal expressions) is at the heart of flight dynamics. For these reasons, a number of conclusions drawn by Ananthkrishnan et al. in their papers[5,6] must be carefully revised and corrected.

References

[1]Bryan, G. H., and Williams, W. E., "The Longitudinal Stability of Aerial Gliders," *Proceedings of the Royal Society, Series A*, Vol. 73, No. 489, 1904, pp. 100–116.
[2]Lanchester, F. W., *Aerodonetics*, Archibald Constable, London, 1908.
[3]Bryan, G. H., *Stability in Aviation*, Macmillan, London, 1911.
[4]Melvill Jones, B., "Dynamics of the Airplane," *Aerodynamic Theory*, edited by W. F. Durand, Vol. 5, Springer, Berlin, 1935, pp. 1–222.

[5]Ananthkrishnan, N., and Unnikrishnan, S., "Literal Approximations to Aircraft Dynamics Modes," *Journal of Guidance, Control, and Dynamics*, Vol. 24, No. 6, 2001, pp. 1196–1203.

[6]Ananthkrishnan, N., and Ramadevi, P., "Consistent Approximations to Aircraft Longitudinal Modes," *Journal of Guidance, Control, and Dynamics*, Vol. 25, No. 4, 2002, pp. 820–824.

[7]McRuer, D., Ashkenas, I., and Graham, D., *Aircraft Dynamics and Automatic Control*, Princeton Univ. Press, Princeton, NJ, 1973, pp. 257, 275, 309–312, 336, 358, 378, 379.

[8]Stevens, B. L., and Lewis, F. L., *Aircraft Control and Simulation*, Wiley, New York, 1992, pp. 207, 216, 217.

[9]McLean, D., *Automatic Flight Control Systems*, Prentice–Hall, New York, 1990, p. 79.

[10]Heffley, R. K., and Jewell, W. F., "Aircraft Handling Qualities Data," NASA CR-2144, Systems Technology, Inc., Hawthorne, CA, Dec. 1972.

Reply by the Authors to G. Mengali

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IN his Technical Comment,[1] Mengali has expressed his opinion on two papers recently published by us and another coauthor.[2,3] Before we consider Mengali's comments, let us briefly summarize the main points presented in our papers.[2,3] In the first paper,[2] we rewrote the linearized small-perturbation equations of aircraft dynamics in second-order form, used a fast–slow decomposition to derive literal approximations to various aircraft dynamic modes, and showed that it was important to account for the fast mode static residual in arriving at the literal approximation for the corresponding slow mode. In the second paper,[3] we developed on the ideas presented in the first paper[2] and derived consistent literal approximations to the longitudinal modes, that is, when the quadratic factors for the short period and phugoid modes from our derivation were multiplied, they satisfied every term in the fourth-order characteristic polynomial for the longitudinal dynamics. Thankfully, Mengali, although unwilling to acknowledge our contributions, does not dispute any of the points just highlighted. Furthermore, it was clearly stated in both the papers[2,3] that the results derived therein were not specific to any select set of aircraft data and that questions regarding the relative numerical significance of the different terms in the literal approximations were irrelevant in this context. Unfortunately, nearly all of Mengali's comments appear to arise from a lack of understanding of this point, coupled with his belief that "it is unlikely that at this stage something significant can be added about such an old problem."[1]

Let us first take up Mengali's comments[1] on the second paper,[3] where we used the ratio of the two timescales T_f/T_s as an order parameter ϵ and then derived consistent literal approximations to zeroth and first order in ϵ. To help establish consistency, all of the terms in the literal approximations and the characteristic polynomial were tagged with a subscript indicating their order in terms of the parameter ϵ. Strangely, Mengali misinterprets "order in ϵ" to mean "order of magnitude," and all his confusion and comments regarding this paper[3] can be seen to stem from this misunderstanding. Once it is clear that "order in ϵ" has nothing to do with the relative numerical value of the term, all of Mengali's remarks[1] about the second paper[3] and the numerical data in his tables become irrelevant.

Let us now return to the several frivolous comments by Mengali on our first paper.[2] For example, note the following points.

1) It is clearly stated that the dimensional stability derivatives are defined in the standard manner as in Refs. 1–3 of Ref. 2, which are standard textbooks in the field, but Mengali prefers another standard.

2) It is clear from the Appendix in Ref. 2 that the moment equations are written in principal axes and, hence, no primed derivatives (as per Mengali's notation) are required to account for the I_{xz} term.

3) Primed derivatives used in Eqs. (6) and (7) of Ref. 2 have been clearly defined following Eq. (7) of that paper, and there can be no cause for confusion.

4) It is also very clearly stated following Eq. (3) in Ref. 2 that rate derivatives for the force equations and derivatives with respect to $\dot{\alpha}$ are not considered for convenience, and not out of any regard for their numerical magnitude.

Mengali's accusation[1] that we ignore phugoid approximations other than that by Lanchester is ridiculous because we refer to the recent paper by Pradeep (Ref. 7 of Ref. 2) on phugoid approximations over the last century and also compare our phugoid approximations with those derived by him. Mengali then goes on to selectively quote us to say that "... they claim that 'previous literal approximations... assumed $\Delta\alpha$ and $\Delta\dot{\alpha}$ to be zero during the phugoid mode.'" However, the correct statement from our paper[2] is as follows: "... previous literal approximations *that did not consider the static residual value of* $\Delta\alpha$, *but* assumed $\Delta\alpha$ and $\Delta\dot{\alpha}$ to be zero during the phugoid mode, essentially ended up with the following equation for the phugoid dynamics...." By deliberately omitting the words in italics, Mengali tries to twist our statement to suit his convenience. Following this, Mengali suggests using a three-degree-of-freedom (3-DOF) approximation for what every one agrees is a 2-DOF phugoid motion and comes up with Eq. (3) in his comment,[1] which can be seen to be incorrect by comparision with Eq. (11) of our paper.[2]

Mengali's comments on our lateral mode approximations are even more bizarre. There is, of course, no sign error in Eq. (19) of our paper[2]; it is just that Mengali, as always, prefers a different sign convention. Again, Mengali is wrong when he suggests that we have claimed the following expression for the spiral eigenvalue to be a new result:

$$\lambda_S = \frac{\mathrm{LHS}\left[S^1_{\mathrm{lat}}\right]}{\omega^2_{\mathrm{DR}}\lambda_R}$$

However, when the new approximation for ω^2_{DR} is substituted in the preceding expression, the resulting formula for λ_S is a new result and is clearly not the same as Mengali's "well known" spiral approximation [Eq. (12) of Ref. 1]. Mengali then argues, without citing any supporting evidence, that the derivative L_β should influence only the dutch-roll damping, but not the frequency. However, a glance at the literature (e.g., Fig. 5.14, p. 201, of Nelson[4]) will establish that his argument is incorrect, and hence, all of his succeeding discussion on the dutch-roll mode is flawed. For example, while trying to factorize the term D in Eq. (21) of his comment,[1] he neglects the Y_β term in his Eq. (19), but fails to notice that our factorization successfully captures all of the terms including the one he ignores.

In conclusion, Mengali's rambling comments[1] have no basis and are devoid of any merit, technical or otherwise.

References

[1]Mengali, G., "Comment on 'Literal Approximations to Aircraft Dynamic Modes' and 'Consistent Approximations to Aircraft Longitudinal Modes,'" *Journal of Guidance, Control, and Dynamics*, Vol. 26, No. 2, 2003, pp. 380–384.

[2]Ananthkrishnan, N., and Unnikrishnan, S., "Literal Approximations to Aircraft Dynamic Modes," *Journal of Guidance, Control, and Dynamics*, Vol. 24, No. 6, 2001, pp. 1196–1203.

[3]Ananthkrishnan, N., and Ramadevi, P., "Consistent Approximations to Aircraft Longitudinal Modes," *Journal of Guidance, Control, and Dynamics*, Vol. 25, No. 4, 2002, pp. 820–824.

[4]Nelson, R. C., *Flight Stability and Automatic Control*, 2nd ed., McGraw–Hill, New York, 1998, p. 201.

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